Tutorial classes Fundamentals of Electrical Engineering

Objective: With the Fundamentals of Electrical Engineeringcourses we have covered the fundamental concepts of electricity. It is time to begin applying these concepts to some common types of circuits. The two common circuit configurations are the “series circuit” and “parallel circuit”.

Purpose:

At the end of this first “TD” you should be able to:

- understand general d.c. circuit theory
- calculate unknown voltages, current and resistances in a series, parallel and series-parallel circuits
- understand voltage division in a series circuit
- understand current division in parallel network
- state and use Kirchhoff’s laws to determine unknown currents and voltages in d.c. circuits
- understand the superposition theorem and apply it to find currents in d.c. circuits

Key Terms:

- Kirchhoff’s Voltage Law
- Series Circuit
- Parallel Circuit
- Voltage divider
- Current divider
- Voltage drop

Exercise 1. Series and parallel circuits

What is the total resistance in the circuit shown in Figures hereafter?

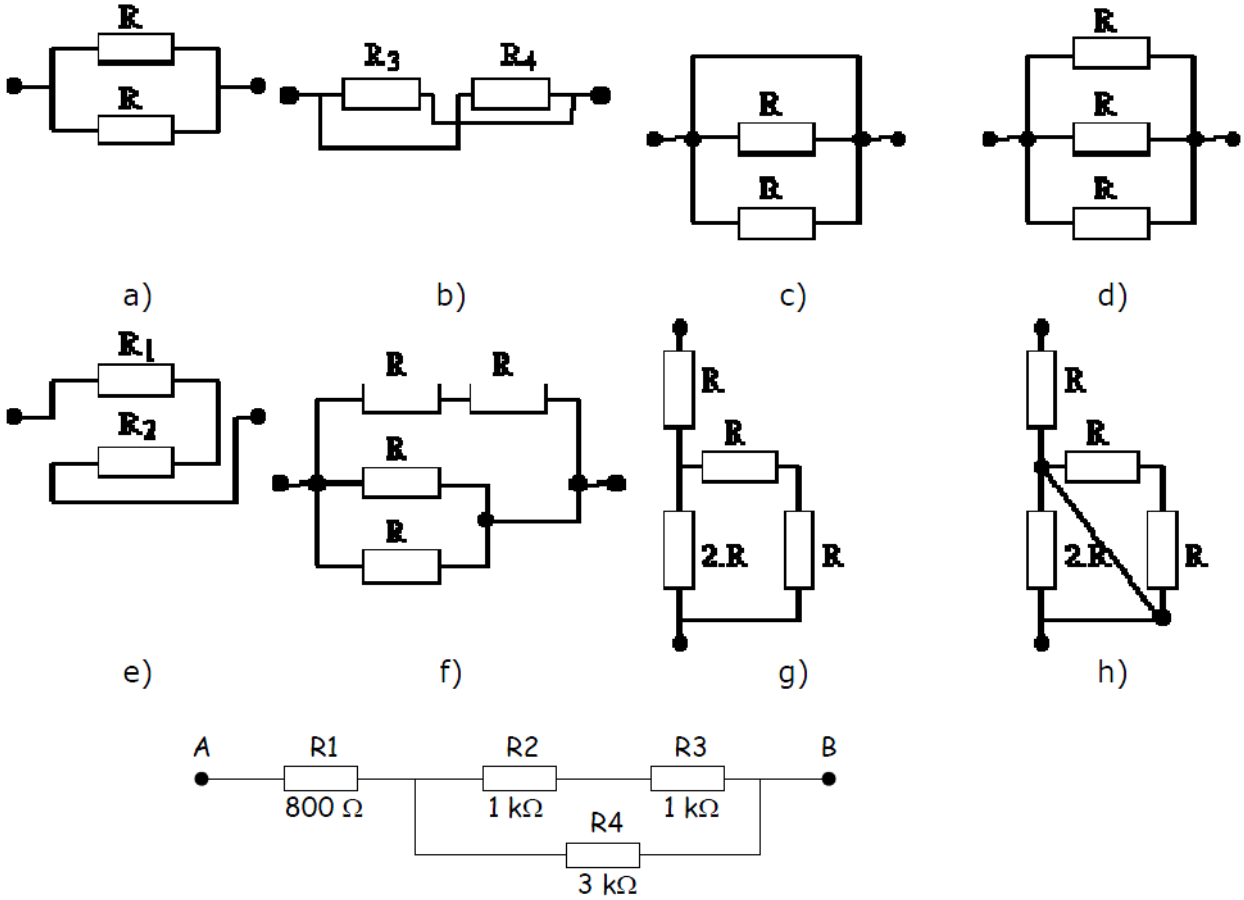

Fig. 1. **Series and parallel circuits**

Exercise 2. KIRCHHOFF'S LOW

Use Kirchhoff's laws to determine the currents in each branch of the network shown in Figure 2. E1 = 10 V, E2 = 5 V, R1 = 15 Ω, R2 = 10 Ω et R3 = 5 Ω.

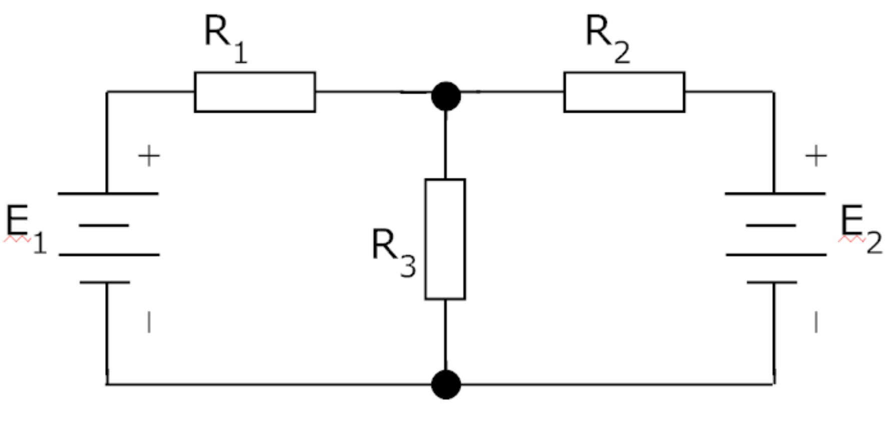

Fig. 2

Procedure

1. Use Kirchhoff's current law and label current directions on the original circuit diagram. The directions chosen are arbitrary, but it is usual, as a starting point, to assume that current flows from the positive terminals of the batteries. The three branch currents can be expressed in terms of I_1 and I_2 only, since the current through R is $I_1 + I_2$.

2. Divide the circuit into two loops and apply Kirchhoff's voltage law to each (the direction chosen does not matter).

3. Solve equations.

Exercise 3. VOLTAGE-DIVIDER

1. Determine the voltage V_1 across R_1, using the V, R_1 and R_2 in the voltage-divider equation (Figure 3).

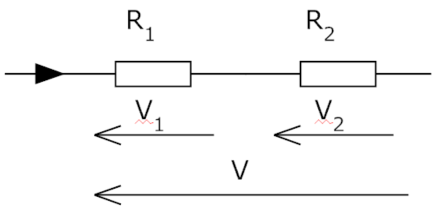

Figure 3

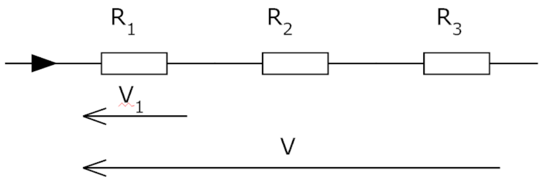

Figure 4

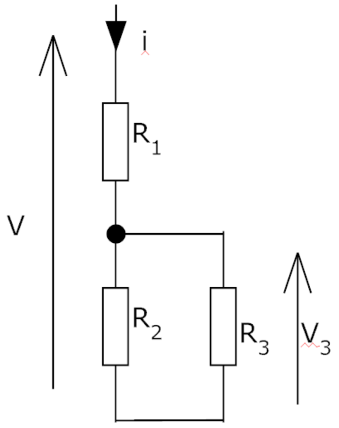

Figure 5

2. Determine the voltage V_1 across R_1, using V, $R_{1,}$ R_2 and R_3 in the voltage-divider equation (Figure 4).
3. Determine the voltage V_3 across R_3, using V, $R_{1,}$ R_2 and R_3 in the voltage-divider equation (Figure 5).

Exercise 4. CURRENT DIVIDER

1. Determine the current i_1, using the i, R_1 and R_2 in the current-divider equation (Figure 6).
2. Determine the current i_1, using the i, R_1, R_2 and R_3 in the current-divider equation (Figure 7).
3. Determine the current i_3, using the i, R_1, R_2 and R_3 in the current-divider equation (Figure 8).

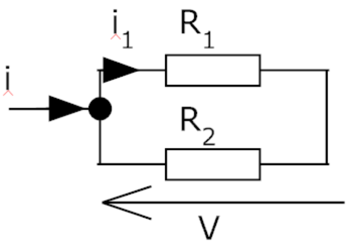

Fig 6

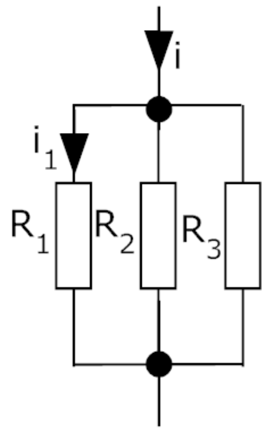

Fig 7

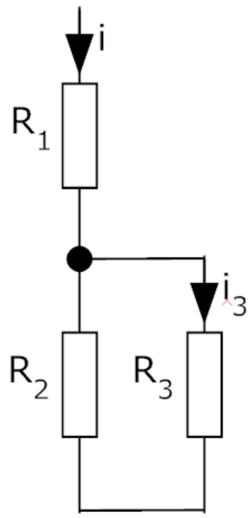

Fig 8

Exercise 5. SUPERPOSITION THEOREM

Figure 9 shows a circuit containing two sources each with their internal resistance. Determine the current in each branch of the network by using the superposition theorem (E_1 = 12 V et E_2 = 9 V.).

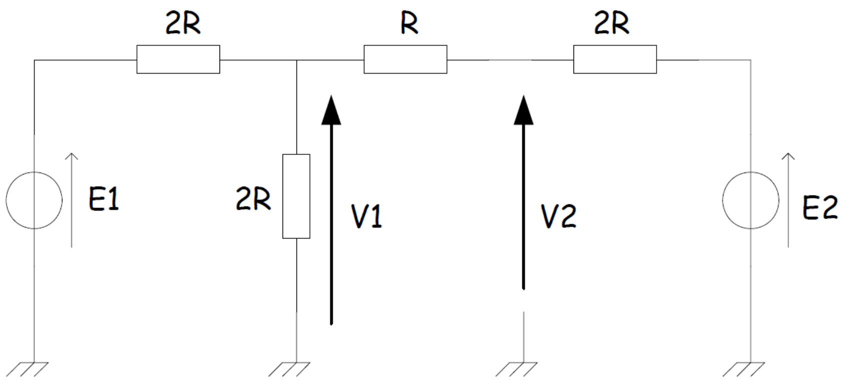

Fig 9

Procedure:

1. Redraw the original circuit with source E_2 removed, being replaced by 2R+R only
2. Label the currents in each branch and their directions and determine their values.
3. Redraw the original circuit with source E_1 removed, being replaced by 2R only.

4. Label the currents in each branch and their directions as shown in and determine their values.
5. Superimpose both circuits.
6. Determine the algebraic sum of the currents flowing in each branch.

Exercise 6. Circuit Analysis

Use the circuit analysis methods to find the voltage across resistors of 10 Ω and 5 Ω (Figure 10)

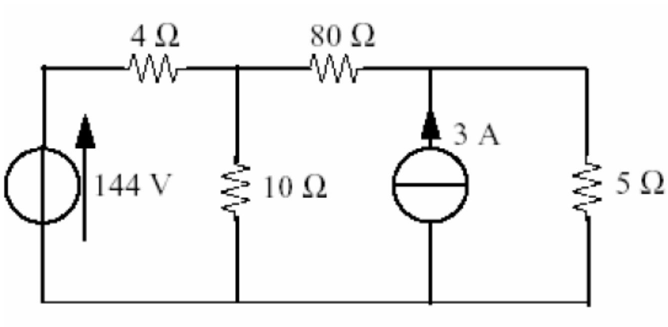

Figure 10

Exercise 7. Application: Acquisition of the speed set point from the handstrap of an electric scooter

If on a thermal scooter a fault of the handstrap of the accelerator is often a break of the cable, forcing the machine to turn at idle speed, what happens on the electric scooters? Could a fault of the accelerator induce a runway of the motor of the machine?
The rotation of the accelerator handstrap of the scooter acts on a potentiometer. The voltage of the cursor, representing the rotation angle of the handstrap is treated, after a conversion of the analogical signal into a digital signal, using the electric control unit. V_D is the set point voltage which allows fixing the desired speed.

Additional resistances (an association of resistance R) allow assuring protection and security of the user in case of break of the cable or wrong connection.
We are going to analyses the principle of acquisition of the speed set point (Figure 11).

V_{CC}=5V; P=4,7 kΩ; R_1=2,2 kΩ, R_2=100 kΩ

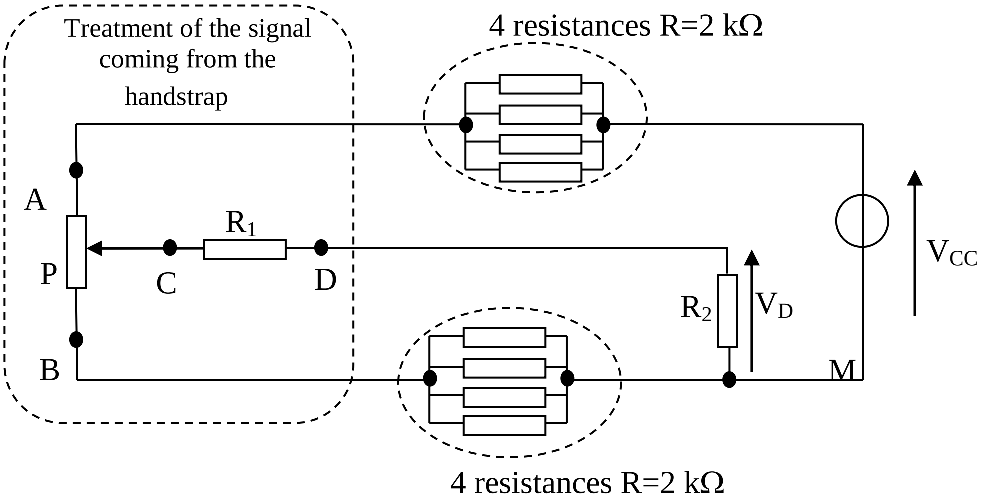

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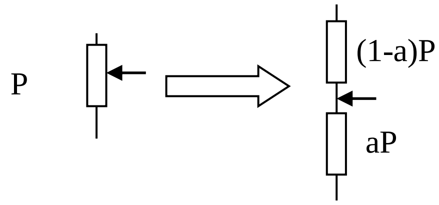
Figure 11

Equivalent circuit of the potentiometer P with $0 \leq a \leq 1$.



1. Calculate the equivalent resistance of the resistance associations R which will be noted Re. Do again an identical schema with all the components (by taking into account the equivalent schema of the potentiometer P).
2. Determine the potential of point C noted V_C as a function of P, R_1, R_2, Re and V_{CC}.
3. What happens to this expression if $R_1+R_2>>a.P+R_e$?
4. Determine the potential of point D noted V_D as a function of R_1, R_2, a, R_e and V_{CC}.
5. The occurrence of a fault on the accelerator control will cause a cut-off or a short circuit.

Fault detection of an electrical connection:

6. We consider a cut-off in the point C or A. Calculate for these two cases, the voltage V_D denoted V_{D1}.
7. We consider a cut-off in the point B. Determine, for this case, the voltage value V_{D2} of V_D as a function of a, P, R_e and V_{CC}.

Detecting a short circuit:

8. We consider a short circuit between the points A and D, D and B then between the points A and B. Calculate V_D and give its variation range.

Purpose:

By the end of this 2nd seminar you should be able to:

- understand Thevenin's theorem and apply a procedure to determine unknown currents in d.c. circuits
- understand Norton's theorem and apply a procedure to determine unknown currents in d.c. circuits
- appreciate and use the equivalence of Thevenin and Norton
- recognize circuit diagram symbols for ideal voltage and current sources

Key Terms:

- Thevenin's Theorem
- Norton's Theorem
- Ideal current source
- Norton current (I_N)
- Norton equivalent circuit
- Norton resistance (R_N)
- Norton's theorem
- Thevenin equivalent circuit
- Thevenin resistance (R_{TH})
- Thevenin's theorem
- Thevenin voltage (E_{TH})

Exercise 1. Equivalent Generators

a) Determine Thevenin's equivalent generator of R_{ch}, between A and B (Figure 1).
N.A.: E = 15V, R_1 = 18 kΩ and R_2 = 3.3 kΩ.
b) Determine Norton's equivalent generator of R_{ch}, between A and B (Figure 2).
N.A.: I = 8mA, R1 = 6.8 kΩ et R2 = 2.2 kΩ.

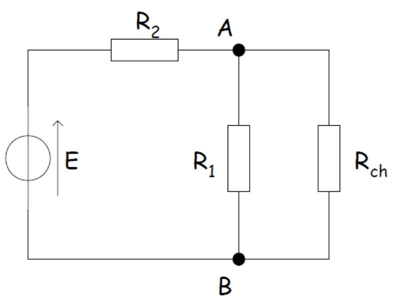


Figure 1.

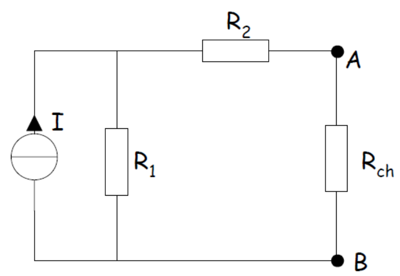


Figure 2

c) Determine the parameters of the Thevenin equivalent generator for the circuit shown in Figure 3, between A and B.

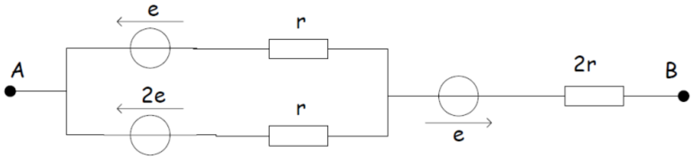


Figure 3

d) Determine Norton's equivalent generator for the circuit shown in Figure 4.
e) Determine Thevenin's equivalent generator for the circuit shown in Figure 4.
f) Determine the current I passing through R and the voltage U across R.

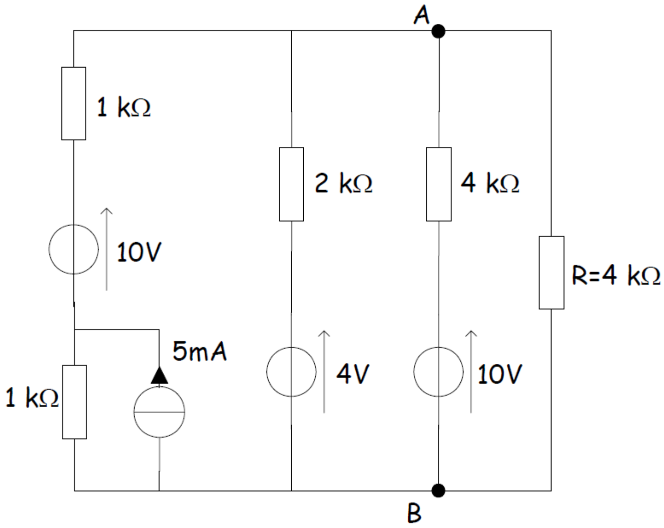


Figure 4

g) Determine Thevenin's equivalent generator for the two terminal circuit AB in Figure 5, taking into account R_1, R_2, I and λ.

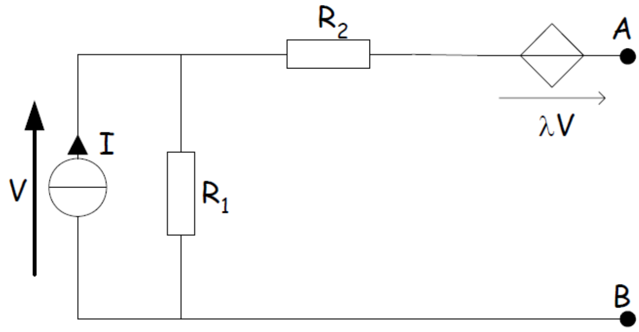


Figure 5

Exercise 2. Power analysis
Use the circuit theory to find the total power dissipated in the resistors in Figure 9 (Hint: the total power is the sum of the individual powers).

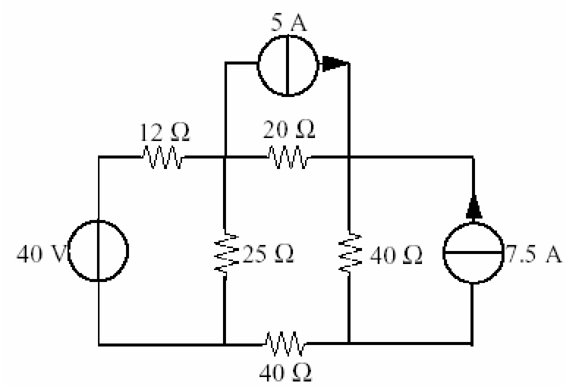


Figure 6

Exercise 3. Millman theorem

a) Calculate using Millman's theorem the E_{th} and R_{th} of R_{ch} (Figure 7).
b) Determine the voltage across R_{ch} (R_{ch} =R).

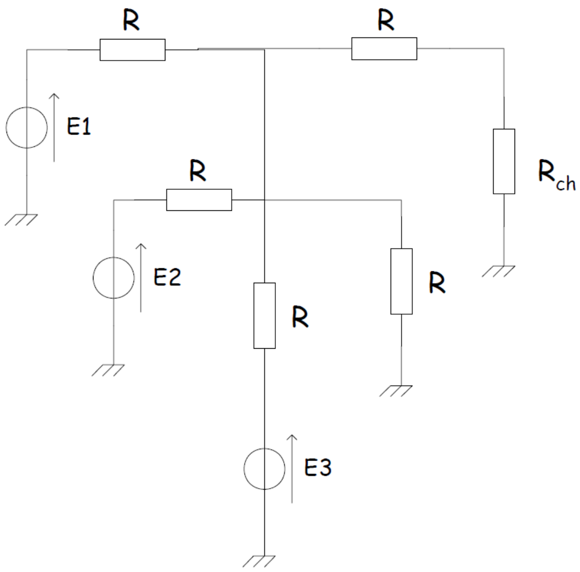


Figure 7

Purpose:

At the end of this 3rd seminar you should be able to:

- define capacitance and state its unit
- describe a capacitor and draw the circuit diagram symbol
- perform simple calculations involving C = Q/V and Q = It
- perform calculations involving capacitors connected in parallel and in series
- describe the charge and discharge characteristics of capacitors
- describe the voltage transitions in a resistive-capacitive DC circuit
- understand the precautions needed when discharging capacitors

Key Terms:

- Capacitance
- Capacitor
- Capacity
- Dielectric
- Electrolyte
- Electrostatic charge
- Farad
- Plates
- Rise curve
- Time constant

Exercise 1. First order DC circuit

An RC circuit is shown in Figure 1. A voltage step E_0 is applied to the circuit during t_1 then the generator E is turned off.

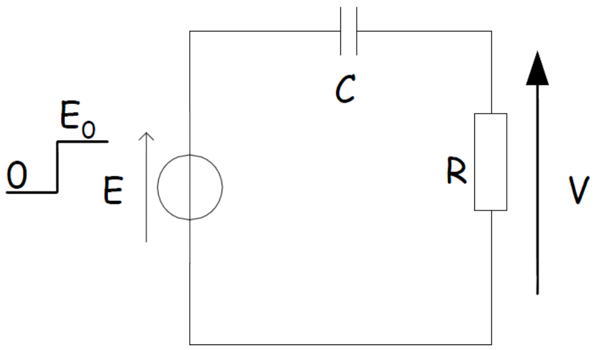


Figure 1

a) Describe the voltage V across resistor R (equation and waveform) if t_1 = 50 ms ?

N.A.: E_0 = 15V, R = 1 kΩ et C = 1 µF.

b) The same question if t_1 = 2ms.

Exercise 2. First order DC circuit

Another RC circuit is shown in figure 2.
The switch K is turned on at t = 0. Describe the voltage evolution U (t) across capacitor C.
N.A.: e = 15V, R = 10 kΩ and C = 100 µF.

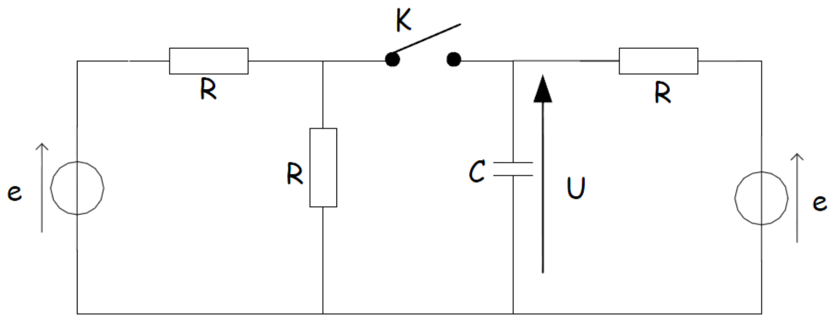


Figure 2

Exercise 3. Second order DC circuit

In the circuit in Figure 3, at the time t = 0, the switch K is turned on, the capacitor is initially discharged.

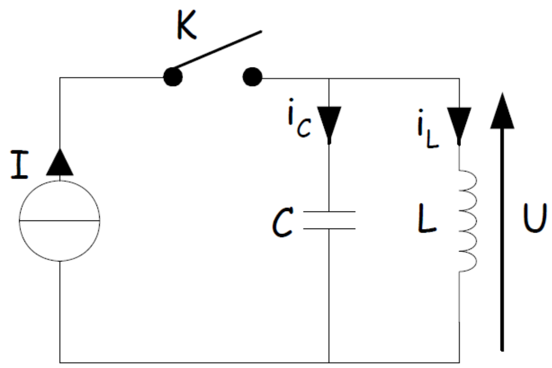


Figure 3

a) Determine the voltage U across the two components. Comments.
b) Determine the currents i_L and i_C through each component.

Exercise 4. Application

A contactor (KM1) is controlled by a bipolar transistor as shown in Figure 4. The transistor acts as a switch allowing current to flow. Its control law is given by the sequence diagram in Figure 5.

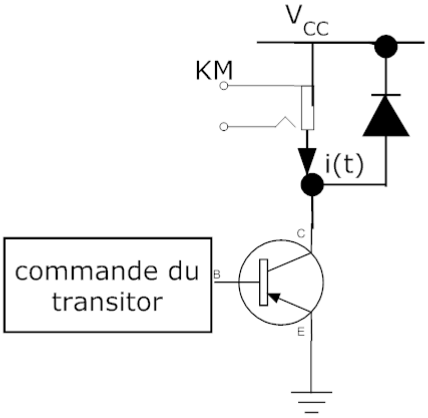


Figure 5

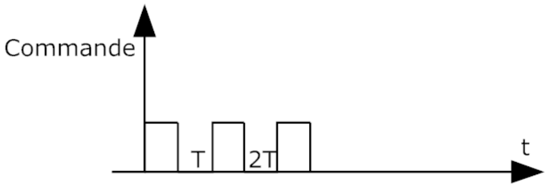


Figure 6

At t = 0, the KM contactor can be modelled by the circuit shown in Figure 6.

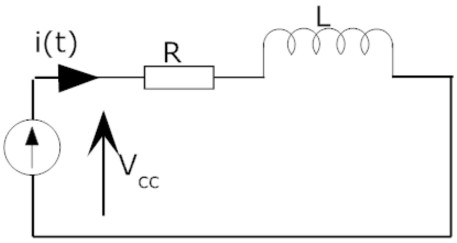


Figure 7

a) Determine and solve the current differential equation considering an initially zero current in the circuit.
b) The contacts close when the current reaches 50% of continuous current. Determining the delay t_e when the contactor is activated.
c) To accelerate the current, we propose placing a resistor R_a in series with R. Justify the reduction of t_e then determine the limit value of R_a in order to always have the contactor switched on.

Exercise 5. Application: Approach of the thermal behaviour of an electric motor

Since 2010 the TTXGP championships (Tourist Trial Xtreme Grand Prix) is increasingly popular not only in the USA but also in Europe. These championships are motorcycle races on a 100% electric circuit. Despite a relatively similar driving behaviour, these modern motorcycles are a far cry from today's thermal motorcycles in terms of speed performance.
The improved performance of these sport motorcycles requires, amongst other things, improving the energy efficiency of their engines.
An electric motor is characterized by ferromagnetic loss, Joule losses in the coils (resistance) and mechanical losses (friction, ventilation etc.), responsible for the temperature increase in the coils and motor body.

Understanding the spread of thermal energy is one of the first steps required to improve engine performance. What causes temperature rises in electric motors?

For simplicity, in the following analyses, we consider that the thermal power P is 100 W.

The simplified thermal diagram will be used during this study. The analogy will be made between thermal elements in this figure and electrical elements:

The thermal resistance (CW^{-1}) is analogous to an electrical resistance (Ω)
The thermal power P (W) is associated with an electrical current (A)
The temperature (θ) is associated with a potential (V)
The temperature difference (Δθ) is associated with a potential difference (V)

One element model:

The ambient temperature is set at $\theta_a = 40$ °C
Rc: thermal resistance °CW. It reflects the heat exchange between two areas, taking into account convection, radiation and conduction. $R_c = 0.3$ °CW^{-1}
C_C: thermal capacity in J °C^{-1}. It reflects the ability of an element to store thermal energy. It is analogous to the capacitor. $C_C = 1000$ J. ° C^{-1}.
θ_C: element temperature
θ_a: ambient temperature

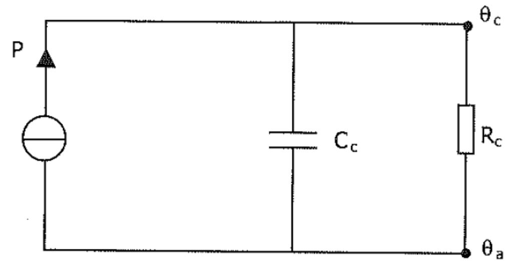


Figure 8

1. Express the differential equation that governs θ_C and resolve. Define the time constant τ_C.
2. Represent θ_C and specify its maximum value.

Two elements model

The previous model does not take into account the evolution of the temperature in the induction-coils which are more sensitive to temperature.
For the next part of the problem, we consider the capacity and the resistance of the induction-coils.

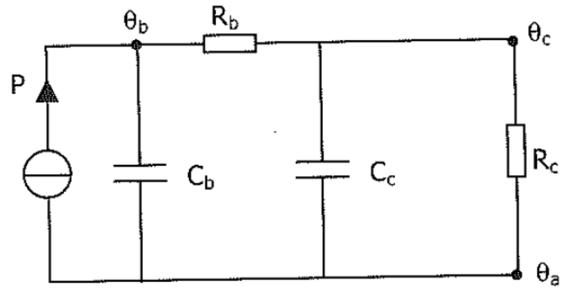


Figure 9

In this figure C_b = 1000 J.°C^{-1} and R_b = 0.15 ° CW^{-1}.

3. Show that one of the equations that reflect the behaviour of the circuit is expressed as follows $P = C_b \frac{\theta_b}{dt} + \frac{\theta_{b-}\theta_c}{R_b}$.
4. Show that another differential equation reflecting the behavior of the whole system is $\frac{R_b \cdot R_c}{R_b + R_c} \cdot C_c \cdot \frac{\theta_c}{dt} + \theta_c \frac{\theta_b \cdot R_c + R_b \cdot \theta_a}{R_b + R_c}$.
5. Taking into account the previous equations, write the differential equation governing θ_b. Solve this differential equation. Represent θ_b and then specify the value of the maximum temperature reached by the inductor.

Purpose:

At the end of this 4th seminar you should be able to:

- define a complex number
- understand the Argand diagram
- perform calculations: addition, subtraction, multiplication and division in Cartesian and polar forms
- describe alternating current (AC)
- describe the phase relationship between RLC components current and voltage
- draw the Fresnel diagram
- calculate any current or impedance value for series/parallel RLC
- compare and contrast the values obtained from current-based and impedance-based phase angle calculations
- compare and contrast apparent power, reactive power and active power

Key Terms:

- Alternating current (AC)
- Average AC power
- Cycle
- Frequency (f)
- Half-cycle
- Instantaneous value
- Magnitude
- Peak value
- Periodic waveform
- Phase
- Phase angle
- Sine wave
- Apparent power
- Reactive power
- Active power
- Impedance
- Inductive reactance (XL)
- Power factor

Exercise 1. Fresnel vector

Express the vector in Figure 1:

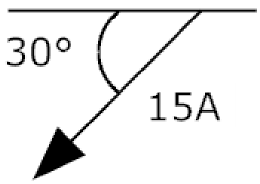


Figure 1

a) polar form (modulus and argument)
b) rectangular form (a+jb)

Exercise 2.
Two vectors are given by: V1 = 63 + j16 and V2 = -5 - j12.
Calculate in polar form:
a) the product value of the vectors
b) the value of V1/V2

Exercise 3.
a) Determine the current's value in resistive, inductive and capacitive circuit elements in Figures 2 to 5.
b) Using the graphical method, find the value of the current supplied by the source and its phase shift relative to the voltage
c) Determine the impedance of the circuits in Figures 2 to 5
d) Calculate the value of the current supplied by the source

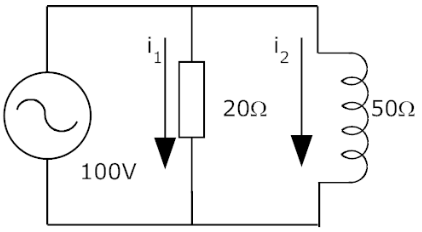


Figure 2

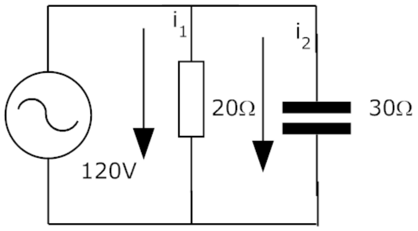


Figure 3

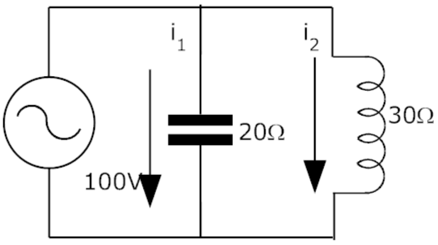


Figure 4

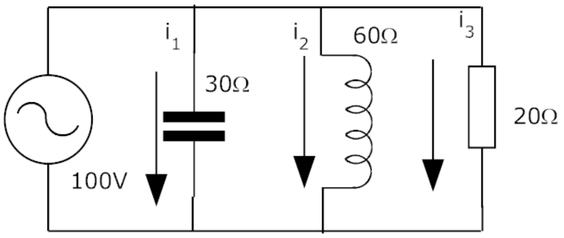


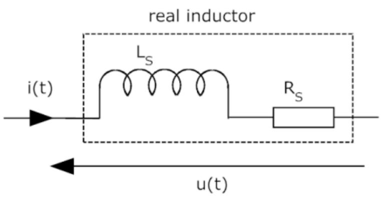
Figure 5

Exercise 4. Application

An impedance Z is described by the expression: Z = 30 - j80
a) What is it made of? Why?
b) Express the impedance in polar form

Exercise 5. Application: Study of a real inductor

The series model of a real inductor is defined by its series resistance R_S and its series inductance L_S. It is possible to define a parallel model equivalent to the series model as shown in Figure 6.



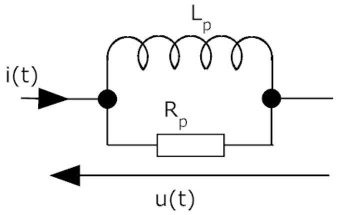


Figure 6

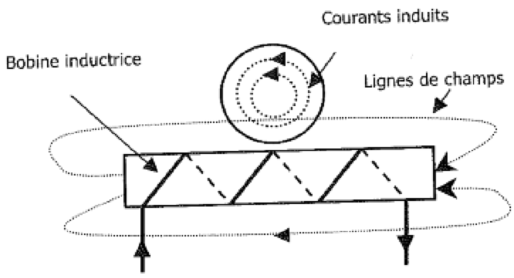
a) Determine the constant R_P and L_P as a function of R_S and L_S. Perform inverse transformation.
b) The inductor is supplied by the voltage u(t)=U2cos(wt). Determine the expression of current i(t) as a function of U, R_S, L_S and ω then U, R_P, L_P and ω.
c) A resistance R_0 assumed to be perfect is placed in series with the rest of the circuit in order to determine the variables R_S and L_S of a real inductor. After supplying the whole system, the voltage across R_0 of the real inductor and the voltage of the assembly are sensed (Figure 7). This procedure is called "three voltmeters method". Draw the Fresnel diagram of the assembly.
d) Deduce the values of the elements of the inductance serial model R_S and L_S given that: the frequency of the voltage supply is f = 50 Hz, U_{R0} = 20 V, U_B = 22.4 U_T = 36 V and R_0 = 10 Ω.
e) Following a similar principle, provide a method to determine R_P and L_P.

Exercise 6. Application: Induction Oven

In France electric heating costs amount to around € 5.2 billion, which means 25% of total heating costs (gas, oil, geothermal, solar, etc). Consequently, heating is a significant share of energy consumption.
It is therefore crucial to use heating-efficient techniques to reduce the impact on the environment.

Presentation

Principle of induction heating



Induction heating is the process of heating an electrically conducting object (usually a metal) by electromagnetic induction, where eddy currents (also called Foucault currents) are generated within the metal and resistance leads to Joule heating of the metal. An induction heater (for any process) consists of an electromagnet, through which a high-frequency alternating current (AC) is passed.

The field of application for induction heating is very wide because it affects both home and industrial uses. Because of the induction coil, an Induction Oven is equivalent to a series circuit with a pure inductor and a resistor.

2. Oven directly powered by a sinusoidal voltage (Figure 7)

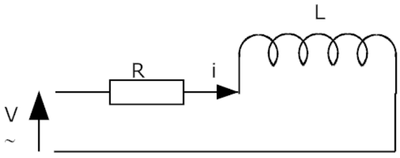


Figure 7

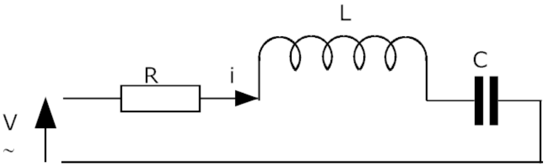


Figure 8

The characteristics of the oven in question are: V = 1845 V, L = $60\mu H$, R = 2.4 Ω. The supply frequency varies around f = 25 kHz.

2.1 Calculate the impedance Z of the oven for f = 25 kHz.
2.2 Deduce the power factor $\cos\varphi$.
2.3 Calculate (rms absorbed current I_0).
2.4 Calculate the active power P_0, reactive Q_0 and apparent S_0 consumed by the oven.
2.5 Draw the active power $P_0 = f(\omega)$. Conclude.

3. Oven + capacitor directly fed by a sinusoidal voltage (Figure 8)

A capacitor C is added in series with the coil and the resistance. The supply voltage is set to 560 V and the frequency f_0 = 25 kHz.

3.1 Calculate the total impedance Z_t to obtain the same current as in question 2.3
3.2 Deduce the power factor $\cos\varphi_t$ and the corresponding phase shifts
3.3 Calculate the capacitance of the capacitors corresponding to phase shifts

For the rest of the exercise we will use the value of the capacitor to which the current is leading the voltage.

3.4 Draw Fresnel diagrams of the voltages.
3.5 Express characteristic $P = f(\omega)$. Conclude.

Exercise 7. Maxwell Bridge

To measure unknown impedance, a bridge impedance method supplied by a sinusoidal voltage source E can be used (Figure 9).

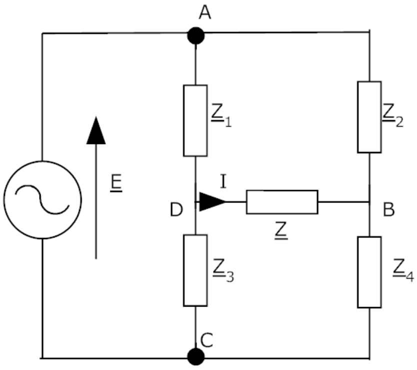


Figure 9

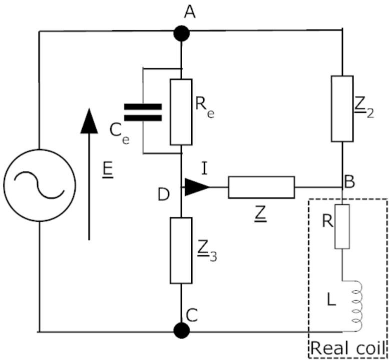


Figure 10

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a) Determine the expression for the current I in the central branch as a function of E, Z_1, Z_2, Z_3, Z_4 and Z.
b) Express the condition on the elements Z_1, Z_2, Z_3 and Z_4 to balance the bridge (cancel the current I).
c) The bridge is used to determine the model parameters of a real series coil (resistance and inductance). This particular structure is called a "Maxwell bridge". The complex impedance of the real coil is denoted Z_x and replaces the previous impedance Z_4. The equivalent admittance of the parallel circuit R_eC_e placed between the points A and D is denoted by Y_e. Determine the complex expressions of Z_x and Y_e.
d) Express L and R as a function of R_2, R_3, R_e and C_e when the bridge is balanced. Determine the numerical values of the elements using: $R_2 = 500\ \Omega$, $R_3 = 50\ \Omega$, $R_e = 125\ \Omega$ and $C_e = 20\mu F$.

Purpose:

At the end of this 5th seminar you should be able to:

- list the four primary filters and identify their frequency response curves
- describe commonly used logarithmic frequency scales
- calculate resonant frequency in an a.c. series circuit
- describe and analyse the low-pass filter operation
- describe and analyse the high-pass filter operation
- define Q-factor
- discuss the relationship between filter Q, band-width, and centre frequency

Key Terms:

- Attenuation
- Band-stop filter
- Bandpass filter
- Band width
- Centre frequency (f_0)
- Decade
- Frequency response
- Gain
- High-pass filter
- Logarithmic scale
- Octave
- Power gain
- Quality (Q)
- Unity gain
- Voltage gain

Exercise 1. Composed Transfer Function

1. Study and draw the asymptotic Bode diagrams of the function:

$$\underline{\ \ }_i = 1 + j\tau_i\omega$$

2. Without studying, extend these results to the transfer function:

$$\underline{\ \ }_i = \frac{1}{1 + j\tau_i\omega}$$

3. Use the previous results to study and draw without calculations the asymptotic Bode diagrams of the transfer function T:

$$\underline{\ \ }_i(j\omega) = k\frac{(1 + j\tau_1\omega)(1 + j\tau_2\omega)}{(1 + j\tau_3\omega)(1 + j\tau_4\omega)}$$

with $\tau 1 < \tau 3 < \tau 2 < \tau 4$.

4. Again without studying, draw the line of the real Bode curves.

Exercise 2. Bandpass filter 2nd order

We will study the structure shown in Figure 1.

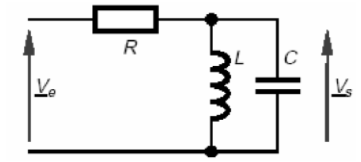

Figure 1 : Bandpass filter 2nd order

1. Determine the transfer function of this filter and write it in following form:

$$\underline{H}(j\omega) = \frac{\underline{V}_s}{\underline{V}_e} = \frac{A_0}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Express the amplification A_0, the Q factor and the angular centre frequency ω_0, as well as their units.

2. Express the gain G (dB) and phase related to this transfer function.

For the asymptotic study of H, we are interested in the behaviour of G and φ at the limits of the frequency domain: ω low and ω high.

3. Show that in each of these areas, the gain behaves like an asymptotic straight line for which the characteristics are provided. Determine the equivalent phases for both frequency ranges.

4. Determine the frequency at which the two asymptotes intersect and the gain for this frequency.

5. Draw the asymptotic Bode diagrams.

We are now interested in the real curve. For this, we analyse the gain curve for the frequency ω_0.

6. Compare the actual gain *(G_0)* to the gain G_i at the intersection of the asymptotes and show that the position of the real curve depends on the position of Q with respect to 1.

7. Draw the shape of the real diagrams for Q<<1 and Q>>1.

8. Give the type of filter.

9. Determine the bandwidth (bandwidth) B at -3 dB.

Exercise 3.

During the restitution of high fidelity music sound, for better results it is important to use the speakers inside a precise frequency range. It is therefore necessary to provide only the signals in the appropriate frequency range using filters: low frequency signals for boomers, high frequency for tweeters.

This association is illustrated on filters used to equip a speaker with 2 channels. The circuit design is shown in Figure 2. In its bandwidth, each speaker is equivalent to a resistance R = 8 Ω. The study also includes the calculation of L and C values.

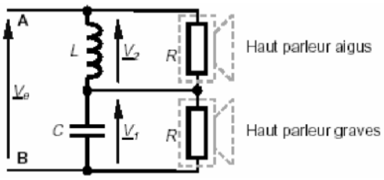

Figure 2: two-way speaker system

Part I: Impedance analyses

1. Determine the expression of the complex impedance Z between the terminals A and B as a function of the angular frequency ω, the signal $\underline{V_e}$, L, R and C.
2. Determine the relationship between R, L and C in order to have the impedance Z equal to the resistance R whatever the angular frequency ω. This condition is called [E1].
3. Deduce the advantage of this condition for power delivered by an amplifier providing the voltage $\underline{V_e}$. For the next part of the exercise, we shall consider the condition [E1] verified.

Part II: First filter analyses

1. Express the transfer function $\underline{H_1}(j\omega) = \frac{\underline{V_1}}{\underline{V_e}}$ and write it in the following form $\underline{H_1}(j\omega) = \frac{1}{1+j\frac{\omega}{\omega_0}}$

then express ω_0 as a function of R and C.

2. Calculate the values of L and C given that the desired filter cut frequency is f_0 = 1 kHz.
3. Briefly analyse and draw the asymptotic and real Bode diagram (G1, gain/ω and φ1, phase/ω) for this filter. Indicate the filter type and check the coherence with the connected speaker.

Part III: Second filter analyses

1. Express the transfer function $\underline{H_2}(j\omega) = \frac{\underline{V_2}}{\underline{V_e}}$.
4. Analyse briefly and draw the asymptotic and real Bode diagram (G_2, gain/ω and φ_2, phase/ω) on the same figure. Indicate the filter type and check the coherence with the connected speaker.
5. Check if the entire frequency domain is covered by the surrounding wall.